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Antonakakis, Nikolaos and Gabauer, David

Webster Vienna Private University, University of Portsmouth, Johannes Kepler University

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Nikolaos Antonakakis[§,†] and David Gabauer[§]

[§]*Department of Business and Management, Webster Vienna Private University, Praterstrasse 23, 1020, Vienna, Austria. Tel: +43-1-2699293-4354. Email: nikolaos.antonakakis@webster.ac.at*

[†]*Economics and Finance Subject Group, University of Portsmouth, Portsmouth Business School, Portland Street, Portsmouth, PO1 3DE, United Kingdom. Tel: +44 (0)23 9284 4261. Email: nikolaos.antonakakis@port.ac.uk*

[‡]*Johannes Kepler University, Department of Economics, Altenbergerstraße 69, 4040, Linz, Austria.*

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Abstract

In this study, we propose refined measures of dynamic connectedness based on a TVP-VAR approach, that overcomes certain shortcomings of the connectedness measures introduced originally by Diebold and Yılmaz (2009, 2012, 2014). We illustrate the advantages of the TVP-VAR-based connectedness approach with an empirical analysis on exchange rate volatility connectedness.

Keywords: Dynamic connectedness, TVP-VAR, Exchange rate volatility

JEL codes: C32; C50; F31; G15

*An online estimation platform of the originally proposed, as well as our refined version of the connectedness approach with replication files, can be accessed here.

1 Introduction

Financial crises are in most of the cases unpredictable. Despite that, the transmission mechanism of shocks related to such crises share certain similarities (Reinhart and Rogoff, 2008). That is why many researchers have developed methodologies in an attempt to capture this transmission process. A notable study, among the many, is by Diebold and Yılmaz (2009, 2012, 2014) who introduced different versions of connectedness procedures based on the notion of forecast error variance decomposition from vector autoregressions (VAR). This VAR-based connectedness methodology has already attracted significant attention by the economic literature, investigating issues such as stock market interdependencies, volatility spillovers, business cycle spillovers and bond yields spillovers (see, inter alia, McMillan and Speight, 2010; Yilmaz, 2010; Bubák et al., 2011; Antonakakis, 2012; Zhou et al., 2012; Antonakakis and Vergos, 2013; Antonakakis and Badinger, 2014; Narayan et al., 2014; Bostanci and Yilmaz, 2015; Diebold and Yılmaz, 2015; Diebold and Yilmaz, 2015).

There have been also several attempts to extent and improve the aforementioned connectedness measures, such as the asymmetric extension by Baruník et al. (2016). Despite that, we argue that there is still room for additional improvements to overcome few of the connectedness measures' shortcomings. In particular, we extend and refine the current connectedness literature by applying a time-varying parameter vector autoregression (TVP-VAR), instead of the currently proposed rolling-window VAR. This improves the methodology provided by Diebold and Yılmaz (2012) substantially, because under our proposed methodology: (1) there is no need to arbitrarily set the rolling window-size, (2) there is no loss of observations and (3) it is not outlier sensitive.

We compare and contrast the originally introduced connectedness measures with our proposed measure of connectedness using an empirical illustration based on the dataset of Antonakakis (2012). We find that, our proposed TVP-VAR-based measure of connectedness adjust immediately to events, while the originally proposed measure based on rolling windows either overreacts (when the rolling-window size is inadequately small) or smoothens the effect out (in the case of setting an inadequately large rolling-window size). A 200-days rolling-window VAR seems to be the closest to the evolution of total connectedness based on the TVP-VAR; which is also in line with the rolling-window size suggested by Diebold and Yılmaz (2012) for daily data. Even in the case of the 200-day rolling window size, the originally proposed measure is still sensitive to extreme outliers.

The remainder of this note is organized as follows. Section 2 describes the data and our proposed methodology. Section 3 illustrates the empirical comparison among the various connectedness measures, and finally, Section 4 concludes this note.

2 Methodology

2.1 TVP-VAR

Our proposed TVP-VAR methodology, extends the originally proposed connectdenss approach of Diebold and Yılmaz (2009, 2012, 2014), by allowing the variances to vary via a stochastic volatility Kalman Filter estimation with forgetting factors introduced by Koop and Korobilis (2014). By doing so, it overcomes the burden of the often arbitrarily chosen rolling-window-size, that could lead to very erratic or flattened parameters, and loss of valuable observations. As such, our approach can also be conducted to examine dynamic connectedness at lower frequencies and limited time-series data.

In particular, the TVP-VAR model can be written as follows,

$$\boldsymbol{Y}_t = \boldsymbol{\beta}_t \boldsymbol{Y}_{t-1} + \boldsymbol{\epsilon}_t \qquad \boldsymbol{\epsilon}_t | \boldsymbol{F}_{t-1} \sim N(\boldsymbol{0}, \boldsymbol{S}_t) \tag{1}$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\nu}_t \qquad \boldsymbol{\nu}_t | \boldsymbol{F}_{t-1} \sim N(\boldsymbol{0}, \boldsymbol{R}_t) \tag{2}$$

where $\boldsymbol{Y}_t$ represents an $N \times 1$ conditional volatilities vector, $\boldsymbol{Y}_{t-1}$ is an $Np \times 1$ lagged conditional vector, $\boldsymbol{\beta}_t$ is an $N \times Np$ dimensional time-varying coefficient matrix and $\boldsymbol{\epsilon}_t$ is an $N \times 1$ dimensional error disturbance vector with an $N \times N$ time varying variance-covariance matrix, $\boldsymbol{S}_t$. The parameters $\boldsymbol{\beta}_t$ depend on their own values $\boldsymbol{\beta}_{t-1}$ and on an $N \times Np$ dimensional error matrix with an $Np \times Np$ variance-covariance matrix.

The time-varying coefficients and error covariances are used to estimate the generalised connectedness procedure of Diebold and Yılmaz (2014) that is based on generalised impulse response functions (GIRF) and generalised forecast error variance decompositions (GFEVD) developed by Koop et al. (1996) and Pesaran and Shin (1998). In order to calculate the GIRF and GFEVD, we transform the VAR to its vector moving average (VMA) representation, based on the Wold representation theorem as follows:

$$\boldsymbol{Y}_t = \boldsymbol{\beta}_t \boldsymbol{Y}_{t-1} + \boldsymbol{\epsilon}_t \tag{3}$$

$$\boldsymbol{Y}_t = \boldsymbol{A}_t \boldsymbol{\epsilon}_t \tag{4}$$

$$\boldsymbol{A}_{0,t} = \boldsymbol{I} \tag{5}$$

$$\boldsymbol{A}_{i,t} = \boldsymbol{\beta}_{1,t} \boldsymbol{A}_{i-1,t} + ... + \boldsymbol{\beta}_{p,t} \boldsymbol{A}_{i-p,t} \tag{6}$$

where $\boldsymbol{\beta}_t = [\boldsymbol{\beta}_{1,t}, \boldsymbol{\beta}_{2,t}, ..., \boldsymbol{\beta}_{p,t}]'$ and $\boldsymbol{A}_t = [\boldsymbol{A}_{1,t}, \boldsymbol{A}_{2,t}, ..., \boldsymbol{A}_{p,t}]'$ and hence $\boldsymbol{\beta}_{i,t}$ and $\boldsymbol{A}_{i,t}$ are $N \times N$ dimensional parameter matrices.

The GIRFs represent the responses of all variables following a shock in variable i. Since we do not have a structural model, we compute the differences between a J-step-ahead forecast where once variable i is shocked and once where variable i is not shocked. The difference can be accounted to the shock in variable i, which can be calculated by

$$GIR_t(J, \boldsymbol{\delta}_{j,t}, \boldsymbol{F}_{t-1}) = E(\boldsymbol{Y}_{t+J} | \boldsymbol{\epsilon}_{j,t} = \boldsymbol{\delta}_{j,t}, \boldsymbol{F}_{t-1}) - E(\boldsymbol{Y}_{t+J} | \boldsymbol{F}_{t-1}) \tag{7}$$

$$\boldsymbol{\Psi}^g_{j,t}(J) = \frac{\boldsymbol{A}_{J,t} \boldsymbol{S}_t \boldsymbol{\epsilon}_{j,t}}{\sqrt{S_{jj,t}}} \frac{\boldsymbol{\delta}_{j,t}}{\sqrt{S_{jj,t}}} \qquad \boldsymbol{\delta}_{j,t} = \sqrt{S_{jj,t}} \tag{8}$$

$$\boldsymbol{\Psi}^g_{j,t}(J) = S_{jj,t}^{-\frac{1}{2}} \boldsymbol{A}_{J,t} \boldsymbol{S}_t \boldsymbol{\epsilon}_{j,t} \tag{9}$$

where J represents the forecast horizon, $\boldsymbol{\delta}_{j,t}$ the selection vector with one on the jth position and zero otherwise, and $\boldsymbol{F}_{t-1}$ the information set until $t-1$. Afterwards, we compute the GFEVD that can be interpreted as the variance share one variable has on others. These variance shares are then normalised, so that each row sums up to one, meaning that all variables together explain 100% of variable's i forecast error variance. This is calculated as follows

$$\tilde{\phi}^g_{ij,t}(J) = \frac{\sum_{t=1}^{J-1} \Psi^{2,g}_{ij,t}}{\sum_{j=1}^{N} \sum_{t=1}^{J-1} \Psi^{2,g}_{ij,t}} \tag{10}$$

with $\sum_{j=1}^{N} \tilde{\phi}^N_{ij,t}(J) = 1$ and $\sum_{i,j=1}^{N} \tilde{\phi}^N_{ij,t}(J) = N$. Using the GFEVD, we construct the total connectedness index by

$$C^g_t(J) = \frac{\sum_{i,j=1, i\neq j}^{N} \tilde{\phi}^g_{ij,t}(J)}{\sum_{i,j=1}^{N} \tilde{\phi}^g_{ij,t}(J)} * 100 \tag{11}$$

$$= \frac{\sum_{i,j=1, i\neq j}^{N} \tilde{\phi}^g_{ij,t}(J)}{N} * 100 \tag{12}$$

This connectedness approach shows how a shock in one variable spills over to other variables. First, we look at the case where variable i transmits its shock to all other variables j, called *total directional connectedness to others* and defined as

$$C^g_{i\rightarrow j,t}(J) = \frac{\sum_{j=1,i\neq j}^N \tilde{\phi}^g_{ji,t}(J)}{\sum_{j=1}^N \tilde{\phi}^g_{ji,t}(J)} * 100 \quad (13)$$

Second, we calculate the directional connectedness variable i receives it from variables j, called *total directional connectedness from others* and defined as

$$C^g_{i\leftarrow j,t}(J) = \frac{\sum_{j=1,i\neq j}^N \tilde{\phi}^g_{ij,t}(J)}{\sum_{i=1}^N \tilde{\phi}^g_{ij,t}(J)} * 100 \quad (14)$$

Finally, we subtract *total directional connectedness to others* from *total directional connectedness from others* to obtain the *net total directional connectedness*, which can be interpreted as the 'power' of variable i, or, its influence on the whole variables' network.

$$C^g_{i,t} = C^g_{i\rightarrow j,t}(J) - C^g_{i\leftarrow j,t}(J) \quad (15)$$

If the net total directional connectedness of variable i is positive, it means that variable i influences the network more than being influenced by that. By contrast, if the net total directional connectedness is negative, it means that variable i is driven by the network.

3 Empirical illustration

In an attempt to exhibit the advantages of our proposed methodology, we use the dataset of the study of Antonakakis (2012) for comparison purposes. Specifically, the dataset consists of the EUR(DM), GBP, CHF and JPY against the USD from January 6th, 1986 till December 30th, 2011. This dataset is split into the following two subperiods: (1) 06.01.1986-31.12.1998 (3,286 observations): pre-Euro period (ERM1) and (2) 04.01.1999-30.12.2011 (3,284 observations): post-Euro period (ERM2). The Deutsche Mark is used as a proxy of the euro for the first subperiod, as it is considered to be the key currency of the ERM1 system. Since these exchange rate series are non-stationary, $I(1)$, we use first log-differences $r_t = ln(y_t) - ln(y_{t-1})$ to get daily exchange returns.[1]

In Figures 1–4, we present the dynamic connectedness measures of our proposed TVP-VAR

[1]Data descriptive statistics can be retrieved from Antonakakis (2012).

approach, along with those based on the traditional rolling-window VAR methodology of Diebold and Yılmaz (2012, 2014). Starting with Figure 1, it can be observed that the dynamic total connectedness index (TCI) based on the TVP-VAR adjusts immediately to events. By contrast, those based on rolling windows, either overreact (when the rolling-window size is inappropriately small, e.g. 100 size), or smoothen the effect out (in the case of setting an inappropriately large rolling-window size, e.g. 300). Nevertheless, it seems that the 200-days rolling-window VAR is closer the the actual evolution of the dynamic TCI based on the TVP-VAR; which is in line with the suggested rolling-window size of Diebold and Yılmaz (2012) based on daily data. Yet, even the 200-day rolling window is sensitive to extreme outliers as illustrated in the upper (lower) panel of Figure 1 during 1990-1992 (2009-2010).

[Insert Figures 1-4 here]

A similar pattern is observed in Figures 2-3, and as a result, the net connectedness measures in Figure 4 based on smallest (largest) rolling-window size does not represent reality well, since they overreact (underreact) to extreme outliers. Hence, our proposed procedure overcomes the aforementioned shortcomings by: (1) adjusting as fast as a small sized rolling-window VAR, yet not overreacting to outliers because of the Kalman Gain (Kalman, 1960) that prevents taking outliers into account, and (2) not smoothing the effects out, as in the case of large window-sized VARs.

The aforementioned differences between the two approaches, can also be observed in Table 1, wherein we present the results of our approach and those of Antonakakis (2012), based on average dynamic connectedness measures.

[Insert Table 1 here]

4 Conclusion

In this study, we extend the dynamic connectedness measures of Diebold and Yılmaz (2014) by employing a time-varying parameter vector autoregressive (TVP-VAR) methodology. The advantage of our proposed TVP-VAR-based connectedness methodology, is that it overcomes certain shortcomings of the aforementioned connectedness measures based on a simple VAR estimated using rolling windows. First, there is no loss of observations in the calculation of the dynamic measures of connectedness resulting from the rolling-window analysis. Second, and more importantly, as there is no rolling-window analysis involved, there is no need to choose, in most cases rather arbitrarily, the sample-size of the rolling-window. Last but not least, it is not outlier sensitive. As such, our methodology provides refined and robust measures of dynamic connectedness. We illustrate the advantages of our TVP-VAR-based connectedness approach with an empirical analysis on exchange rate volatility connectedness.

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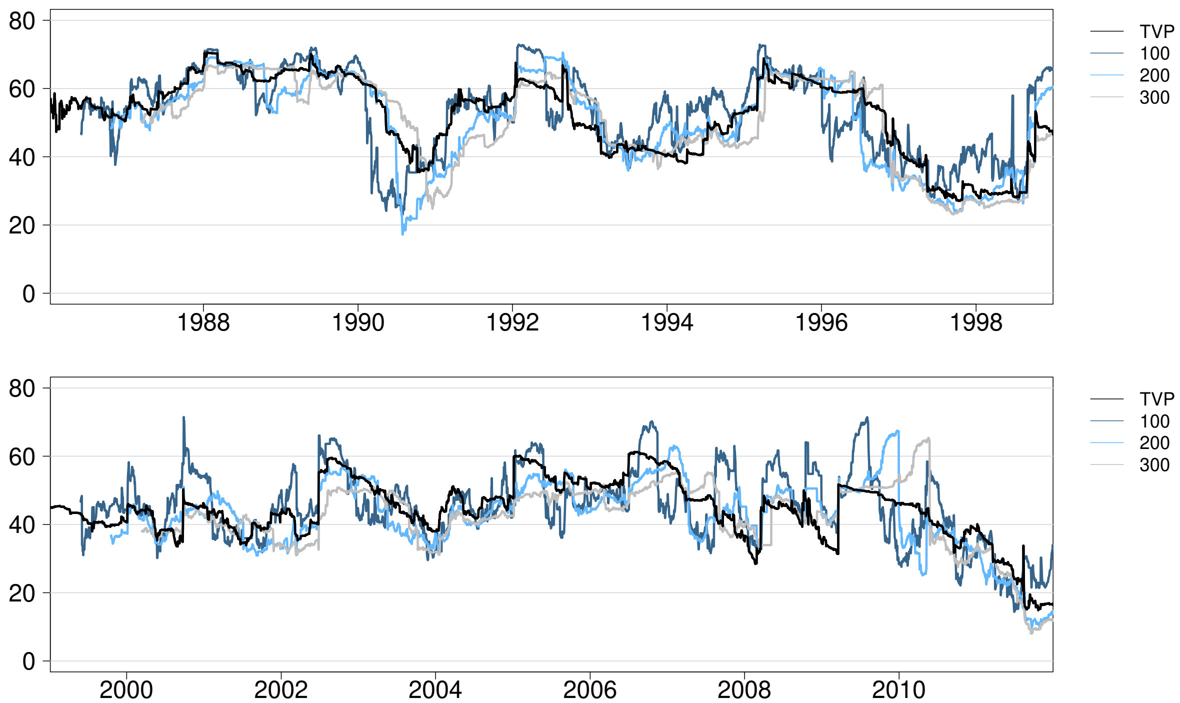
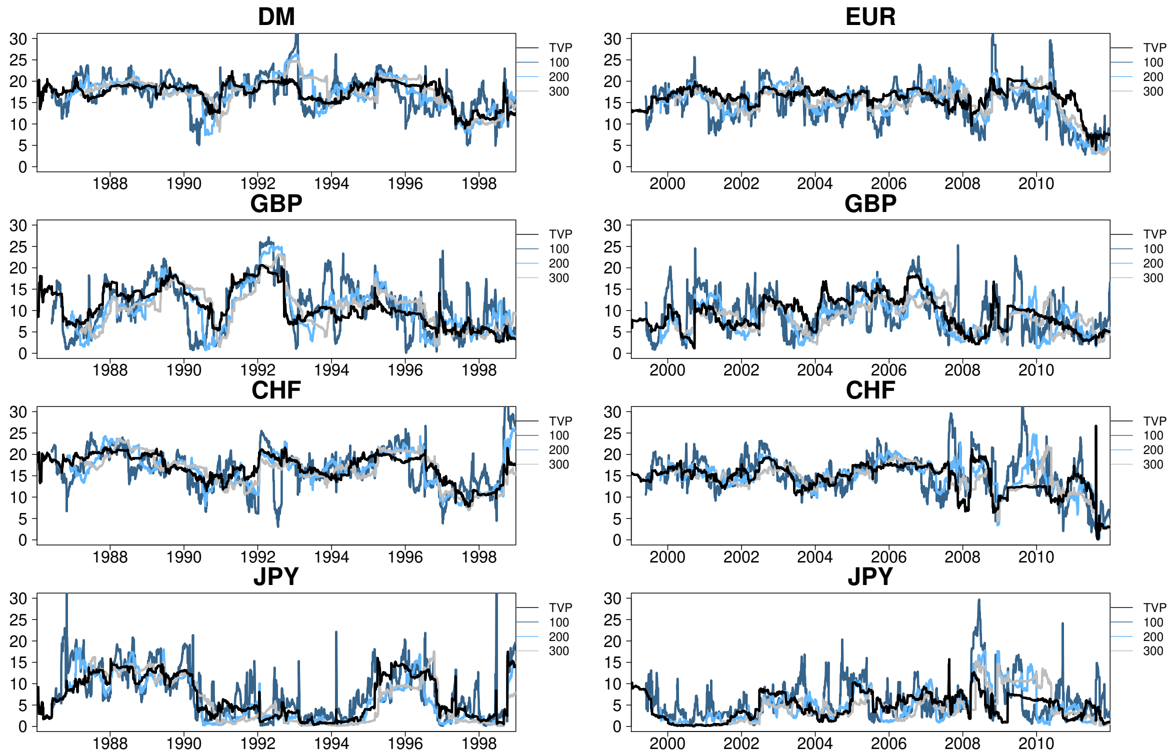
Figure 1: Total connectedness



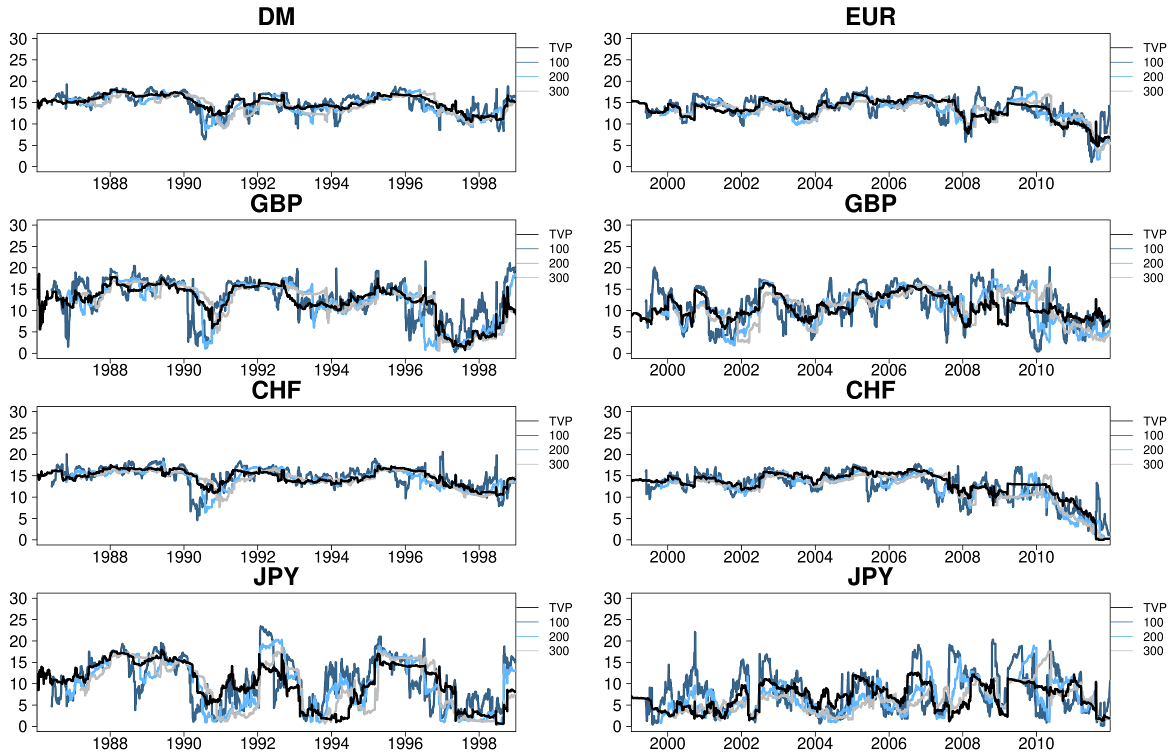
Notes: Black line (TVP) denotes total connectedness based on TVP-VAR. Dark-grey, light-blue and light-grey lines denote total connectedness based on 100, 200 and 300 days rolling window VAR.

Figure 2: Directional connectedness FROM four markets



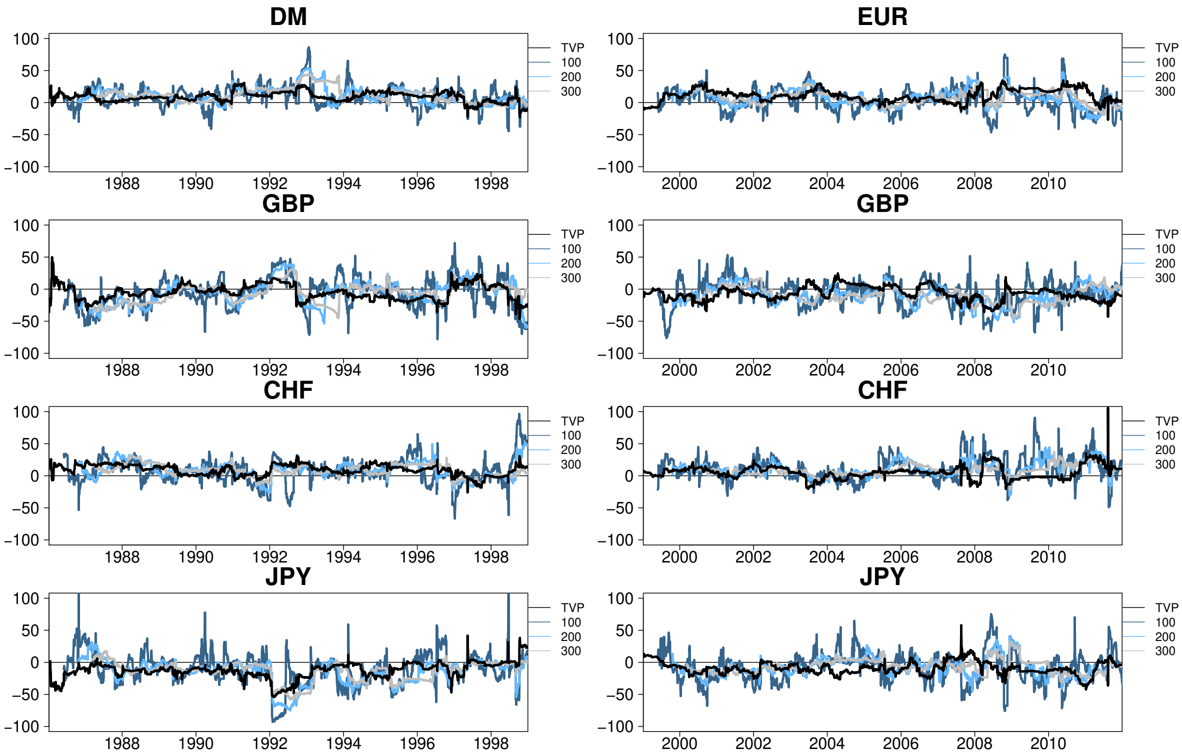
Notes: Black line (TVP) denotes total connectedness based on TVP-VAR. Dark-grey, light-blue and light-grey lines denote total connectedness based on 100, 200 and 300 days rolling window VAR.

Figure 3: Directional connectedness TO four markets



Notes: Black line (TVP) denotes total connectedness based on TVP-VAR. Dark-grey, light-blue and light-grey lines denote total connectedness based on 100, 200 and 300 days rolling window VAR.

Figure 4: Net connectedness



Notes: Black line (TVP) denotes total connectedness based on TVP-VAR. Dark-grey, light-blue and light-grey lines denote total connectedness based on 100, 200 and 300 days rolling window VAR.

Table 1: Dynamic Connectedness Table

Panel a: Pre-Euro (06.01.86-31.12.98)					
	From (j)				
To(i)	DM CV	GBP CV	CHF CV	JPY CV	from others
DM	42.4	17.2	32.0	8.4	57.6
GBP	21.7	52.4	18.9	7.1	47.6
CHF	33.1	15.6	42.7	8.6	57.3
JPY	14.4	9.5	15.2	60.8	39.2
Contribution to others	69.1	42.3	66.2	24.1	201.7
Contribution including own	111.5	94.7	108.9	84.9	TCI
Net connectedness	11.5	-5.3	8.9	-15.1	50.4
Panel b: Post-Euro (04.01.99-30.12.11)					
	From (j)				
To(i)	EUR CV	GBP CV	CHF CV	JPY CV	from others
EUR	47.3	14.9	32.7	5.1	52.7
GBP	19.8	57.5	15.5	7.2	42.5
CHF	30.8	10.9	51.9	6.3	48.1
JPY	9.1	6.8	10.6	73.5	26.5
Contribution to others	59.8	32.7	58.9	18.7	170.0
Contribution including own	107.0	90.1	110.7	92.2	TCI
Net connectedness	7.0	-9.9	10.7	-7.8	42.5

Notes: Values reported are variance decompositions for estimated VAR models for the conditional volatility (CV) obtained from the DCC model in Table 2. Variance decompositions are based on 10-step-ahead forecasts. In both periods, a VAR lag length of order 4 was selected by the BIC.

Panel a: Pre-Euro (06.01.86-31.12.98)					
	From (j)				
To(i)	DM CV	GBP CV	CHF CV	JPY CV	from others
DM	39.9	17.8	32.2	10.1	60.1
GBP	21.9	51.4	19.7	7.0	48.6
CHF	32.2	16.1	41.2	10.5	58.8
JPY	14.8	9.6	16.1	59.5	40.5
Contribution to others	68.9	43.5	68.0	27.6	208.0
Contribution including own	108.9	94.9	109.2	87.0	VSI
Net connectedness	8.9	-5.1	9.2	-13.0	52.0
Panel b: Post-Euro (04.01.99-30.12.11)					
	From (j)				
To(i)	EUR CV	GBP CV	CHF CV	JPY CV	from others
EUR	46.5	17.0	30.8	5.6	53.5
GBP	22.4	56.4	15.6	5.5	43.6
CHF	32.2	12.6	47.9	7.3	52.1
JPY	9.8	7.0	10.9	72.3	27.7
Contribution to others	64.4	36.6	57.3	18.5	176.8
Contribution including own	111.0	93.0	105.2	90.8	VSI
Net connectedness	11.0	-7.0	5.2	-9.2	44.2

Notes: Values reported are variance decompositions for estimated TVP-VAR models for the conditional volatility (CV) obtained from the DCC-GARCH model. Variance decompositions are based on 10-step-ahead forecasts. In both periods, a TVP-VAR lag length of order 1.

A Appendix

A.1 Technical Appendix

The TVP-VAR is represented as follows,

$$\boldsymbol{Y}_t = \boldsymbol{\beta}_t \boldsymbol{Y}_{t-1} + \boldsymbol{\epsilon}_t \qquad \boldsymbol{\epsilon}_t | \boldsymbol{F}_{t-1} \sim N(\boldsymbol{0}, \boldsymbol{S}_t)$$
$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\nu}_t \qquad \boldsymbol{\nu}_t | \boldsymbol{F}_{t-1} \sim N(\boldsymbol{0}, \boldsymbol{R}_t)$$

where $\boldsymbol{Y}_t$ represents an $Nx1$ conditional volatilities vector, $\boldsymbol{Y}_{t-1}$ is an $Np \times 1$ lagged conditional vector, $\boldsymbol{\beta}_t$ is an $N \times Np$ dimensional time-varying coefficient matrix and $\boldsymbol{\epsilon}_t$ is an $N \times 1$ dimensional error disturbance vector with an $N \times N$ time varying variance-covariance matrix, $\boldsymbol{S}_t$. The parameters $\boldsymbol{\beta}_t$ depend on their own values $\boldsymbol{\beta}_t$ and on an $N \times Np$ dimensional error matrix with an $Np \times Np$ variance-covariance matrix.
The prior parameters $\boldsymbol{\beta}_0$ and $\boldsymbol{S}_0$ are set equal to the results of a VAR based on the first 200 days.

$$\boldsymbol{\beta}_0 \sim N(\boldsymbol{\beta}_{OLS}, \Sigma^{\beta}_{OLS})$$
$$\boldsymbol{S}_0 = \boldsymbol{S}_{OLS}.$$

The Kalman Filter estimation, whereby $\kappa_2 = 0.99$, starts with

$$\boldsymbol{\beta}_t | \boldsymbol{Y}_{1:t-1} \sim N(\boldsymbol{\beta}_{t|t-1}, \boldsymbol{\Sigma}^{\beta}_{t|t-1})$$
$$\boldsymbol{\beta}_{t|t-1} = \boldsymbol{\beta}_{t-1|t-1}$$
$$\hat{\boldsymbol{R}}_t = (1 - \kappa_2^{-1}) \boldsymbol{\Sigma}^{\beta}_{t-1|t-1}$$
$$\boldsymbol{\Sigma}^{\beta}_{t|t-1} = \boldsymbol{\Sigma}^{\beta}_{t-1|t-1} + \hat{\boldsymbol{R}}_t$$

The multivariate EWMA procedure for $\boldsymbol{S}_t$ is updated in every step, while κ_1 is set equal to 0.99. If we would assume constant variances we would set this parameter to unity.

$$\hat{\boldsymbol{\epsilon}}_t = \boldsymbol{Y}_t - \boldsymbol{Y}_{t-1} \boldsymbol{\beta}_{t|t-1}$$
$$\hat{\boldsymbol{S}}_t = \kappa_1 \boldsymbol{S}_{t-1|t-1} + (1 - \kappa_1) \hat{\boldsymbol{\epsilon}}'_t \hat{\boldsymbol{\epsilon}}_t$$

$\boldsymbol{\beta}$ and Σ^{β} are updated by

$$
\begin{aligned}
\boldsymbol{\beta}|\boldsymbol{Y}_{1:t} \sim & N(\boldsymbol{\beta}_{t|t}, \boldsymbol{\Sigma}^{\beta}_{t|t}) \\
\boldsymbol{\beta}_{t|t} = & \boldsymbol{\beta}_{t|t-1} + \boldsymbol{\Sigma}^{\beta}_{t|t-1}\boldsymbol{Y}'_{t-1}(\hat{\boldsymbol{S}}_t + \boldsymbol{Y}_{t-1}\boldsymbol{\Sigma}^{\beta}_{t|t-1}\boldsymbol{Y}'_{t-1})^{-1}(\boldsymbol{Y}_t - \boldsymbol{Y}_{t-1}\hat{\boldsymbol{\beta}}_{t|t-1}) \\
\boldsymbol{\Sigma}^{\beta}_{t|t} = & \boldsymbol{\Sigma}^{\beta}_{t|t-1} + \boldsymbol{\Sigma}^{\beta}_{t|t-1}\boldsymbol{Y}'_{t-1}(\hat{\boldsymbol{S}}_t + \boldsymbol{Y}_{t-1}\boldsymbol{\Sigma}^{\beta}_{t|t-1}\boldsymbol{Y}'_{t-1})^{-1}(\boldsymbol{Y}_{t-1}\boldsymbol{\Sigma}^{\beta}_{t|t-1})
\end{aligned}
$$

Then we update the variances, $\boldsymbol{S}_t$, by the EWMA procedure

$$
\begin{aligned}
\hat{\boldsymbol{\epsilon}}_{t|t} = & \boldsymbol{Y}_t - \boldsymbol{Y}_{t-1}\boldsymbol{\beta}_{t|t} \\
\boldsymbol{S}_{t|t} = & \kappa_1 \boldsymbol{S}_{t-1|t-1} + (1-\kappa_1)\hat{\boldsymbol{\epsilon}}'_{t|t}\hat{\boldsymbol{\epsilon}}_{t|t}
\end{aligned}
$$